MATH 1650 SOLVING EQUATIONS INVOLVING LOGARITHM FUNCTIONS

1.

$$\begin{aligned}
\log_{117}(1-3x) &= \log_{117}\left(x^2-3\right) \\
1-3x &= x^2-3 && \text{Same base, equate arguments.} \\
x^2+3x-4 &= 0 \\
(x+4)(x-1) &= 0 && \text{Factor.} \\
x &= -4, 1
\end{aligned}$$

We find both answers check in the original equation, so we keep both solutions.

2.

$$\begin{aligned}
2-\ln(t-3) &= 1 \\
\ln(t-3) &= 1 \\
e^1 &= t-3 && \text{Rewrite as an exponential equation.} \\
t &= e+3
\end{aligned}$$

Substitution $t = e+1$ into the original equation checks.

3.

$$\begin{aligned}
\log_6(x+4)+\log_6(3-x) &= 1 \\
\log_6\left[(x+4)(3-x)\right] &= 1 && \text{Product Rule for Logs} \\
6^1 &= (x+4)(3-x) && \text{Rewrite as an exponential equation.} \\
x^2+x-6 &= 0 \\
(x+3)(x-2) &= 0 && \text{Factor.} \\
x &= -3, 2
\end{aligned}$$

Both solutions, $x=-3$ and $x=2$, check in the original equation.

4.

$$\begin{aligned}
\log_7(1-2t) &= 1-\log_7(3-t) \\
\log_7(1-2t)+\log_7(3-t) &= 1 && \text{Gather the Logs on one side.} \\
\log_7[(1-2t)(3-t)] &= 1 && \text{Product Rule for Logs.} \\
7^1 &= (1-2t)(3-t) && \text{Rewrite as an exponential equation.} \\
2t^2-7t-4 &= 0 \\
(2t+1)(t-4) &= 0 && \text{Factor.} \\
t &= -\tfrac{1}{2}, 4
\end{aligned}$$

When checking the answers in the original equation, we find that $t=4$ produces a negative inside the logarithm: $\log_7(-7) = 1-\log_7(-1)$. Hence, $t=4$ is an **extraneous** solution so we only keep $t=-\frac{1}{2}$.

5.

$$\begin{aligned}
\log_2(x+3) &= \log_2(6-x)+3 \\
\log_2(x+3)-\log_2(6-x) &= 3 && \text{Gather the Logs on one side.} \\
\log_2\left(\frac{x+3}{6-x}\right) &= 3 && \text{Quotient Rule for Logs.} \\
2^3 &= \frac{x+3}{6-x} && \text{Rewrite as an exponential equation.} \\
8(6-x) &= x+3 && \text{Multiply both sides of the equation by } (6-x). \\
x &= 5
\end{aligned}$$

We find $x = 5$ checks in the original equation.

6.

$$\begin{aligned}
1+2\log_4(t+1) &= 2\log_2(t) \\
1 &= 2\log_2(t)-2\log_4(t+1) && \text{Gather the Logs on one side.} \\
1 &= 2\log_2(t)-2\left(\tfrac{1}{2}\log_2(t+1)\right) && \text{Change of Base for Logs.} \\
& && \log_4(t+1)=\tfrac{\log_2(t+1)}{\log_2(4)}=\tfrac{1}{2}\log_2(t+1) \\
1 &= \log_2\left(t^2\right)-\log_2(t+1) && \text{Power Rule for Logs.} \\
1 &= \log_2\left(\frac{t^2}{t+1}\right) && \text{Quotient Rule for Logs.} \\
\frac{t^2}{t+1} &= 2 && \text{Rewrite as an exponential equation.} \\
t^2 &= 2(t+1) && \text{Multiply both sides by } (t+1). \\
t^2-2t-2 &= 0 \\
t &= 1\pm\sqrt{3} && \text{Quadratic Formula.}
\end{aligned}$$

Note the solution $t = 1-\sqrt{3} < 0$. Hence if substituted into the original equation, the term $2\log_2\left(1-\sqrt{3}\right)$ is undefined, which makes $t = 1-\sqrt{3}$ an extraneous solution. Our only answer is $t = 1+\sqrt{3}$.